Teaching for Understanding: "Roadmapping" DCPS Standards

In order to teach content standards, teachers and others involved in implementing standards in DCPS classrooms need strategies for identifying just what the standards mean and how to impart them effectively to students. Among other strategies, DCPS has adopted a process to create classroom "roadmaps" for its standards, adapted from the work of Grant Wiggins and Jay McTighe, authors of *Understanding by Design.*

While teachers throughout the district will be involved in this methodology, curriculum writing teams have developed the following exemplar roadmaps in reading/English language arts and mathematics for each grade. A blank template also is available for teacher use (and can be downloaded from the DCPS Web site).

Here is a brief explanation of the roadmapping process:

The first step is to "unwrap" the standards for the purpose of determining the critical concepts and skills contained within them. The second step is to identify the "Big Ideas," or enduring understandings, from the identified concepts and skills. The Big Ideas offer a broad perspective, purpose, and rationale. They explain what students are doing and why or how their work today relates to larger ideas. The Big Ideas are what we want students to discover and remember long after instruction ends.

With these Big Ideas clearly in mind, we formulate "Essential Questions" to share with students at the inception of an instructional unit. Essential Questions are provocative and important to discuss and debate; they engage inquiry, run to the heart of the subject, and raise important conceptual or philosophical issues. These questions are meant to probe and stimulate student reflection and rethinking. They also guide educators in the selection of lessons and activities that will advance student understanding of the unwrapped concepts and skills. The goal is for students to be able to reflect on the Essential Questions and to state the Big Ideas *in their own words* by the conclusion of an instructional unit.

Each roadmap includes a learning context that embeds the learning of practical skills and concepts within "Engaging Scenarios" that draw learners in and make them think, reflect, and decide. Engaging Scenarios bring the outside world into the classroom and provide the context for applying the standard. The purpose of including Engaging Scenarios is to demonstrate that learning can be made much more interesting than just flipping pages, viewing static graphics, and responding to multiple choice questions at the end of an advisory. They should motivate the student to get involved and answer the question "Why are we doing this?"

"Performance Tasks" are meaningful activities that require a range of behaviors, employing both lower and higher order thinking skills, and enable students to demonstrate their knowledge of the standard. In identifying Performance Tasks, we look for authentic activities, exercises, or problems that require students to create a response to a problem and then explain or defend it rather than simply select an answer from a ready-made list.

"Performance Assessments" are guides that make it clear to students exactly what they are expected to know and do. They fit hand in glove with the performance tasks and should cause students to demonstrate the degree to which they have mastered the standard that drives each roadmap.

Roadmapping standards is a powerful practice that is central to the work of teachers, the students, classroom activities, and the norms and cultures of classrooms across the district. For further information on each component of the roadmap, see "Making StandardsWork," by the Center for Performance Assessment.

"STANDARDS IN ACTION" ROADMAPS – MATHEMATICS

STANDARD:

SUPPORTING/CONNECTING STANDARDS:

GRADE:	UNIT NAME & ADVISORY PERIOD:

Concepts:

Skills:

Suggested Big Ideas (3):

Suggested Essential Questions (4–6):

Suggested Performance Tasks:

Suggested Engaging Scenarios:

Suggested Performance Assessment(s):

Suggested Resources:

Textbook Materials:

Supplementary Materials:

"STANDARDS IN ACTION" ROADMAPS – MATHEMATICS

STANDARD:
AI.P.9. Demonstrate facility in symbolic manipulation of polynomial and rational expressions* by rearranging and collecting terms, factoring [e.g., $a^2 - b^2 = (a+b)(a-b)$, $x^2+10x+21=(x+3)(x+7)$, $5x^4+10x^3-5x^2=5x^2(x^2+2x-1)$], identifying and canceling common factors in rational expressions, and applying the properties of positive integer exponents.

SUPPORTING/CONNECTING STANDARDS:
AI.P.8. Add, subtract, and multiply polynomials with emphasis on first- and second- degree polynomials.

GRADE:	UNIT NAME & ADVISORY PERIOD:
Algebra I	Chapter 10: Polynomials and Factoring (Fourth Advisory)

Concepts:

- Polynomial
- Rational expressions
- Terms
- Factoring
- Common factors (in rational expressions)
- Properties of positive integer exponents

Skills:

- Demonstrate (facility in symbolic manipulation of polynomial and rational expressions)
- Rearrange (terms)
- Collect (terms)
- Factor (polynomials)
- Identify (common factors in rational expressions)
- Cancel (common factors in rational expressions)
- Apply (properties of positive integer exponents)

Suggested Big Ideas:

- An algebraic expression is a collection of variables and real numbers. The most common type of algebraic expression is the polynomial.
- Polynomials can model real-life situations in three ways: constant model *(y = a)*, linear model *(y = ax + b)*, and quadratic model *(y = ax^2 + bx + c = 0)*.
- Factoring (writing a polynomial as a product) is an important tool for solving equations and for simplifying fractional expressions.
- To multiply polynomials, we can use special product patterns, FOIL, areas of rectangle, vertical and horizontal methods, and the distributive property of multiplication.

Suggested Essential Questions:

- What is a polynomial? How do we write polynomials in standard form?
- In what ways can polynomials model real-life situations?
- How do we use special products to multiply polynomials?
- What is factoring? Describe its importance.

Suggested Performance Tasks:

- Classify a polynomial by degree and by the number of terms.
- Add and subtract polynomials.
- Use algebra tiles to add, subtract, multiply, and factor polynomials.
- Multiply polynomials using special product patterns, FOIL, areas of rectangle, vertical, and horizontal methods.
- Use the distributive property to multiply and factor a polynomial.
- Use different types of factoring to factor polynomials expressions and write them in factored form:
    a. Common monomial
    b. Difference of two squares
    c. Perfect square trinomial
    d. Quadratic trinomial
    e. Sum and difference of two cubes
    f. By grouping
- Solve polynomial equations by factoring. Apply factoring skills to write models for real-life situations.

Suggested Engaging Scenarios:

A. Modeling Dimensions with Polynomials. You are working as an architect in the Department of Housing. Your director asked you to form a team of young architects who will build a miniature of a warehouse to be used as a model in the department's future projects. Your team is required to build a box with its specific dimensions. Here is a sample of steps that you and your group could start with.

Work in a group of four students. You will need:

- Rectangular pieces of paper that are not square
- A ruler

Each of you should build a box and write expressions for its dimensions as outlined in steps 1–8.

1. Fold a rectangular piece of paper into 16 equal parts. Always fold toward the front of the piece of paper.
2. Hold your paper with the longer side facing you, and fold the outer flaps back in.
3. Fold in the four corners so the edges line up with the horizontal fold lines.
4. Fold back two vertical strips in the middle, one to the left and the other to the right, to "lock" the corners.
5. Lift up gently to form your box.
6. Unfold your box. Measure the length and the width of the original piece of paper. Also measure the width of the vertical strips that you folded back.
7. What do *x* and the region outlined in red represent in terms of the box?
8. Use measurements from Step 6 to write expressions for the length, width, and height of the box in terms of *x*.
9. Compare expressions with those of other members of your group. How are the expressions alike? How are they different?

Suggested Performance Assessments:

1. Classifying Monomials

	Polynomial	*Degree*	*Classified by Degree*	*Classified by Number of Terms*
a.	6			
b.	$-2x$			
c.	$3x + 1$			
d.	$-x^2 + 2x - 5$			
e.	$4x^3 - 8x$			
f.	$2x^4 - 7x^3 - 5x + 1$			

2. You are designing a stained-glass window. The border is made of red glass and is *x* inches wide. The interior uses other colors of glass. Write a polynomial expression for the amount of red glass you will use. Then use the expression to find out how much red glass would be used for a 2-inch border and for a 3-inch border.

3. You sell hot dogs for $1.00 each at your concession stand at a baseball park and have about 200 customers. You want to increase the price of a hot dog. You estimate that you will lose three sales for every $.10 increase. The following equation models your hot dog sales revenue *R*, where *n* is the number of $.10 increases:

Concession stand revenue model: R = (1 + 0.1n)(200 – 3n)

   a. To find your revenue from hot dog sales, you multiply the price of each hot dog sold by the number of hot dogs sold. In the formula above, what does (1 + 0.1n) represent? What does (200 – 3n) represent?
   b. How many times would you have to raise the price by $.10 to reduce your revenue to zero? Make a graph to help you find your answer.
   c. Decide how high you should raise the price to make the most money. Explain how you got your answer.

Writing:

Write your own multi-step problem about selling a product like the one in Exercise 2. Include a model that shows the relationship between price and number of items sold. Explain what each factor in the model represents.

Suggested Resources:

Textbook Materials:

- *Algebra 1, Larson, et al.,* McDougal Littell, pp. 574–640.

Supplementary Materials:

- *Explorations and Projects Book in Algebra I, Geometry, and Algebra II*, McDougal Littell, pp. 123–124.
- *TI-83/84 Plus* and *TI-89 Manual* to accompany *Stats: Modeling the World* (Bock, Velleman & De Veaux), by Patricia Humphrey, Pearson Addison Wesley, pp. 25–33; 58–63; 67–73.
- Fathom Dynamic Data Software Version 2 (KCP Technologies, Inc.); visit www.keypress.com/fathom for details.
- The Geometer's Sketchpad Dynamic Geometry Software for Exploring Mathematics (KCP Technologies, Inc.); visit Sketchpad Resource Center at www.keypress.com/sketchpad for details.
- www.springboard/collegeboard.com
- www.keypress.com
- www.titutorials.com (for graphing calculators online)

* Rational expressions (operations, standard, and roadmap) can be found in Chapter 11, Rational Equations and Functions.

"STANDARDS IN ACTION" ROADMAPS – MATHEMATICS

STANDARD:

AI.P.11. Perform basic arithmetic operations with rational expressions and functions.

SUPPORTING/CONNECTING STANDARDS:

AI.P.10. Divide polynomials by monomials with emphasis on first- and second-degree polynomials.
AI.P.9. Demonstrate facility in symbolic manipulation of polynomial and rational expressions by rearranging and collecting terms, factoring [e.g., $a^2 - b^2 = (a + b)(a - b)$, $x^2 + 10x + 21 = (x + 3)(x + 7)$, $5x^4 + 10x^3 - 5x^2 = 5x^2 (x^2 + 2x - 1)$], identifying and canceling common factors in rational expressions, and applying the properties of positive integer exponents.
AI.N.3. Calculate and apply ratios, proportions, rates, and percentages to solve a range of consumer and practical problems.

GRADE:	UNIT NAME & ADVISORY PERIOD:
Algebra I	Chapter 10: Polynomials and Factoring (Fourth Advisory)

Concepts:
- Basic arithmetic operations
- Rational expressions
- Rational functions

Skills:
- Perform (basic arithmetic operations with rational expressions and functions)

Suggested Big Ideas:
- A rational expression is the quotient of two polynomials.
- Rational expressions can model real-life situations as ratios, rates, percents, and probabilities.
- We can use rational functions to model the change in the ratios of two quantities over a period of time.

Suggested Essential Questions:
- What is a rational expression?
- How do we add, subtract, multiply, and divide rational expressions?
- How do we simplify and use rational expressions as real-life models?
- How do we use rational functions to model and solve real-life problems?

Suggested Performance Tasks:

- Use ratio, proportion, and percent to solve real-life problems.
- Use direct and inverse variation to model real-life situations.
- Simplify and use rational expressions to find geometric probabilities.
- Add and subtract rational expressions with (a) like denominators and (b) unlike denominators.
- Multiply and divide rational expressions.
- Solve rational equations and graph rational functions.

Suggested Engaging Scenarios:

A. The Capture-Recapture Method. You are a neophyte biologist assigned to work at the Department of Agriculture. On the first day of your job, the director asked you to demonstrate how to estimate the number of wild deer on a certain mountain. How will you perform your task? Here is a sample of steps that you and your group could start with:

You need a large coffee can, a bag of dried beans, and a marker. Pour the bag of beans into the can. Take out a handful of beans, mark them, and record how many you marked. Put the marked beans back into the can. Mix the beans thoroughly. Take out another handful, or *sample*, of beans. Record the number of marked beans and the total number of beans in your sample. *Questions:*

1. Let B = the unknown number of beans in the can. What is the fraction of beans that are marked? *(Your answer will involve B.)*
2. What fraction of beans in your sample was marked? How do you think this fraction relates to the fraction from question 1?
3. Ayita marked 23 beans. Then she took a sample of 12 beans and found three marked. Use Ayita's method and the fractions you found in Questions 1 and 2 to estimate the number of beans in your coffee can.
4. What is the importance of this activity? When can we apply this capture-recapture method?

B. Exploring Rectangles with Equal Areas. You are asked to design a rectangular box with an area of 36 m^2. You are requested to show some samples first before they accept your work. How will you model your design to get their approval? Here is a sample of steps that you could start with. You need 36 algebra tiles and graphed paper.

1. Arrange the 36 tiles to form a rectangle. What are the length and the width of your rectangle?
2. Use the algebra tiles to form as many rectangles as you can with area 36. Copy and extend the table to record the length (l) and the width (w) of each rectangle.
3. Graph the data pairs (l, w) from your table in a coordinate plane.
4. (a) Could a rectangle with area 36 be 4.5 units wide? If so, what would the length be? (b) Draw a smooth curve passing through the points you graphed. Explain why it makes sense to connect the points.
5. Can a rectangle be 0 units long? What happens as the length of the rectangle gets very small? Adjust your graph if necessary.
6. Write an equation relating l and w. How does the length of a rectangle affect its width if the area is constant?

Suggested Performance Assessments:

A. *Project: Miniature Room. Suppose you are working as a model builder and you are asked to construct a miniature room. How will you build your project?*
   - *Objective:* Make a 3-D scale model of a room.
   - *Materials:* Cardboard box, poster board, construction paper, scissors, glue, other decorative materials such as felt or scraps of cloth.

   Investigation:
   1. Measure the dimensions of a room and the position of fixed objects in the room such as doors and windows. Draw diagrams for later reference.
   2. Set up a table to record all necessary measurements and computations. The table below shows some sample items. In the first two columns, fill in the measurements you took in Step 1. Then choose some furnishings you would like to include in your model. Measure their dimensions, and add those objects and their actual measurements to your table.

Object	Actual length (in m.)	Proportion to use (in cm.)	Length in model
Length of the room			
Width of the room			
Height of the room			
Distance from corner to door			
Distance from the bottom of the window to the floor			
Length of the table			

   3. Decide on an appropriate scale to use for your model. If you want to use a box for the walls of the room, then set up the scale so that the length or the width of the box corresponds to the length or the width of the room. (You can trim the sides of the box to match the other dimensions.) For each object, write and solve a proportion to find its length in the model. Record the proportions and the lengths you find in your table.
   4. Use your diagrams and your table to complete a 3-D scale model of the room, including some furnishings.

   Present Your Results: Display your model with a poster. On the poster, include your table and explanation of your procedures including how you solved your proportions.

B. *Research/Survey. Algebra at Work.* Interview at least one working professional *(examples of working professionals are electrician, model builder, college recruiter, service industry careers, and sports reporter)* and ask the following questions:
   1. As _____, do you use rational expressions in your work?
   2. Could you give a simple and specific example of how a rational expression is used?
   3. Should job applicants in your area have an Algebra course in their background?
   4. What other skills are important for employees?

Suggested Resources:

Textbook Materials:

- *Algebra 1, Larson, et al.,* McDougal Littell, pp. 642–705.

Supplementary Materials:

- *Explorations and Projects Book in Algebra I, Geometry, and Algebra II*, McDougal Littell, pp. 123–124.
- *TI-83/84 Plus* and *TI-89 Manual* to accompany *Stats: Modeling the World* (Bock, Velleman & De Veaux), by Patricia Humphrey, Pearson Addison Wesley, pp. 25–33; 58–63; 67–73.
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- The Geometer's Sketchpad Dynamic Geometry Software for Exploring Mathematics (KCP Technologies, Inc.); visit Sketchpad Resource Center at www.keypress.com/sketchpad for details.
- www.springboard/collegeboard.com
- www.keypress.com
- www.titutorials.com (for graphing calculators online)