Grade 8 Unit 5 Standards-Based Worksheet

District of Columbia Public Schools — Mathematics

STANDARD

8.G.1. Analyze, apply, and explain the relationship between the number of sides and the sums of the interior and exterior angle measures of polygons.

Concepts:
- sides
- sums
- interior angle measures
- exterior angles measures
- polygons

Skills:
- analyze (relationship of sides and sums)
- apply (relationship of sides and sums)
- explain (relationship of sides and sums)

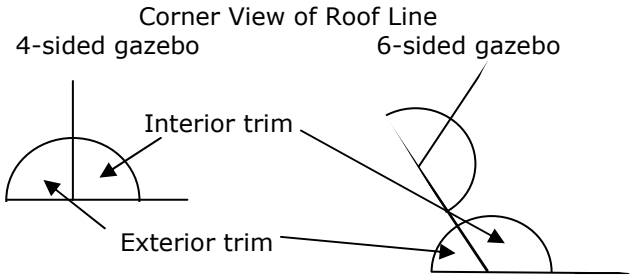
Big Ideas:
- Relationships exist between the angles and sides of polygons.
- Analyzing patterns leads to the discovery of mathematical relationships.

Essential Questions:
- How are interior angles, exterior angles, and the number of sides of a polygon related?
- How are interior angles of polygons measured?
- How are exterior angles of polygons measured?

Engaging Scenario:
You supervise the carpentry shop for a business that has just begun to sell garden gazebos. You plan to produce three gazebo styles: 4-sided, 6-sided, and 8-sided.

Each gazebo has ornamental trim both on its exterior roof line and on its interior roof line. The trim will be cut in sectors from a small circular sheet of decorative wood.

The diagram below shows the corner view of a 4-sided and 6-sided gazebo with trim in place:



1) The trim will be placed in the exterior angles of the gazebo. How many circular sheets of decorative wood will you need to purchase in order to produce the exterior sectors for each style of gazebo?

2) The trim will also be placed in the interior angles of the gazebo. How many circular sheets of decorative wood will you need to purchase in order to produce the interior sectors for each style of gazebo?

PERFORMANCE TASKS	THINKING SKILLS (LOT → HOT)*	PERFORMANCE TASK ASSESSMENT (PROFICIENT CRITERIA)
Students illustrate the placement of the interior and exterior trim in complete diagrams of each of the three gazebo styles.	illustrate (analysis) LOT	Students illustrate a square (or rectangle), a hexagon, and an octagon with the trim pieces drawn in a fashion similar to the given diagram.
Students identify that the trim pieces represent interior and exterior angles of polygons.	identify (knowledge) LOT	Students correctly identify the exterior trim pieces as corresponding to exterior angles and interior trim pieces as corresponding to interior angles in the polygons.
Students calculate the sums of the angles represented by the sectors of trim for each gazebo style, first for the exterior sectors and then for the interior sectors.	calculate (application) LOT	Student calculations should be accurate. They may be based on measuring the angles with protractors or applying the relationship that the sum of the exterior angles in a polygon is 360° and that the sum of the interior angles in a polygon is $[180(n-2)]^{\circ}$, where n is the number of sides.
Students infer how many circular trim sheets will be needed for each gazebo's exterior trim.	infer (analysis) HOT	Students infer that each gazebo needs one circular trim sheet because the sum of the exterior angles in a polygon is 360°, the number of degrees in a circle.

Students decide how many circular trim sheets are needed for each gazebo's interior trim.	decide (evaluation) HOT	Students decide that one, two, and three sheets of trim need to be purchased for each of the four, six, and eight-sided gazebos respectively. They base their decision on determining how many times 360 divides into each of the gazebo's interior angle sum measurements.

*See Bloom's Taxonomy of Thinking Skills to determine higher order thinking skills (HOT) or lower order thinking skills (LOT). The goal is to create tasks that employ higher order thinking skills.

Standard Assessment:

1) Molly formed three polygons—a triangle, a rectangle, and a pentagon—with string. She calculated the sum of the measures of the interior angles for each polygon and entered her data in the chart shown to the right.
   a) What is the sum of the measures of the interior angles of a hexagon?
   b) What is the sum of the measures of the interior angles of an octagon?
   c) How many sides does an unnamed polygon have if the sum of the measures of the interior angles is 2340°?
   d) Explain how you would find the sum of the measures of the interior angles of an *n*-sided polygon.

(MCAS, Grade 8, 2002, Question #29)

Type of Polygon:	Number of Sides:	Sum of the Measures of the Interior Angles:
Triangle	3	180°
Rectangle	4	360°
Pentagon	5	540°
Hexagon	6	?
Octagon	8	?
Unnamed Polygon	?	2340°
n-sided Polygon	*n*	?

2) The sum of the interior angles of a polygon is the same as the sum of its exterior angles. What type of polygon is it?
   a) quadrilateral
   b) hexagon
   c) octagon
   d) decagon

(CA Standards Test, Geometry Released Test Questions, 2005, Question #27)

3) If the measurement of the exterior angle of a regular polygon is 120°, how many sides does the polygon have?
   a) 3
   b) 4
   c) 5
   d) 6

(CA Standards Test, Geometry Released Test Questions, 2005, Question #29)

Resources:

Textbook Materials:

Connected Mathematics, Grade 6, Shapes and Designs (entire unit).
Springboard Mathematics Middle School Mathematics I, The College Board, What's My Name.
Springboard Mathematics Geometry, The College Board, Plenty of Polygons.

Supplementary Materials:

http://regentsprep.org/Regents/math/poly/Lpoly1.htm (Website for general help on polygons.)
Larson, Boswell, Stiff. *Geometry: Concepts and Skills*, McDougal Littell, Chapter 8, 2004, Polygons and Area (DCPS high school geometry text which introduces this topic.)
http://www.algebralab.org/lessons/less.aspx?file=Geometry_AnglesSumPolygons.xml (An excellent graphic representation of the geometric rationale for the interior angle sum formula.)

Grade 8 Unit 5 Standards-Based Worksheet

District of Columbia Public Schools — Mathematics

STANDARD

8.G.2. Demonstrate an understanding of the relationships of angles formed by intersecting lines, including parallel lines cut by a transversal.

Concepts:

- angles
- intersecting lines
- parallel lines
- transversal

Skills:

- demonstrate (an understanding of relationships of angles)

Big Ideas:

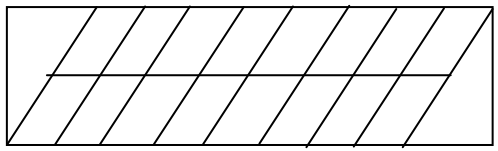
- Line and angle relationships are idealized in geometry, but are evident in the world around us.
- Relationships exist between angles formed by intersecting lines.
- Parallel lines do not intersect.
- Relationships exist between angles formed by parallel lines and a transversal line that intersects them.

Essential Questions:

- How are intersecting lines and the angles they create related to one another?
- Why are vertical angles congruent?
- Why are the relationships between angles formed by parallel lines and a transversal line that intersects them important?

Engaging Scenario:

You are drafting a design for an open-air parking lot for a very large shopping mall. In order to maximize parking, cars will be parked in "angle" parking spaces running in two different directions, as shown below.



In one double row of parking, there will be an angle of 105° between a line in front of each parked car and the parking space line running past the driver's seat. In the next double row of parking, there will be an angle of 75° between a line in front of each parked car and the parking space line running past the driver's seat. As shown in the diagram, at each end of the double row of parking spaces there is a triangular space.

Design work is well behind schedule, so you need to make the fewest number of angle templates possible to complete the blueprint. What angle templates are needed to make a blueprint of the parking spaces and the triangular spaces? Explain your answer and be sure to include a drawing that shows at least two rows of parking spaces with angle relationships and measures labeled on the diagram. Then, test your design using the number of angle templates you specified.

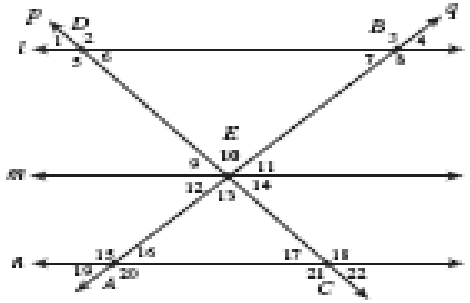
PERFORMANCE TASKS	THINKING SKILLS (LOT → HOT)*	PERFORMANCE TASK ASSESSMENT (PROFICIENT CRITERIA)
Students convert the diagram of the parking lot into a sketch showing at least two rows of parking spaces.	convert (comprehension) LOT	Students convert the diagram showing at least two double rows of parking spaces, each with eight parking spaces on a side, and triangular spaces on each end of the double row. Angle relationships should approximate those in the description.
Students classify the angle relationships on their sketch and label them.	classify (analysis) LOT label (knowledge) LOT	On the sketch, students should classify and label as congruent at least: • two pairs of vertical angles. • two pairs of alternate interior angles. • Three pairs of corresponding angles. They should also classify and label: • A pair of supplementary angles that are *not* same-side interior angles. • A pair of supplementary angles that are same-side interior angles. • A pair of complementary angles.
Students speculate how many angle templates they would need to create the features of this parking lot.	speculate (synthesis) HOT	Students speculate that given that all angles are either a right angle, or congruent or supplementary to 75°, that the parking spaces and triangular ends can be designed with either a 105° or a 75° template, coupled with a right angle.

Students test their ideas on parking lot design, using the number of templates that they have specified.	test (evaluation) HOT	Student test designs should be evaluated for conformance with the description, although the design need not be to a realistic scale and double parking rows need not be separated by a realistic distance for traffic.

*See Bloom's Taxonomy of Thinking Skills to determine higher order thinking skills (HOT) or lower order thinking skills (LOT). The goal is to create tasks that employ higher order thinking skills.

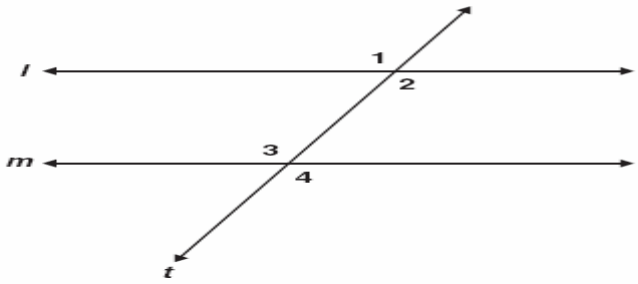
Standard Assessment:

1) In the figure below, the following statements are true:
   - Lines *l, m,* and *n* are parallel.
   - The perpendicular distance from line *m* to line *n* is less than the perpendicular distance from line *m* to line *l.*
   - Lines *m, p,* and *q* intersect at point *E.*
   - Lines *p* and *q* are perpendicular.
   - Angles 6 and 7 are congruent



Are triangles DEB and AEC similar? Explain your answer.
Are triangles DEB and AEC congruent? Explain your answer,
List the 8 angles whose measures are equal to that of <2.
(http://www.doe.mass.edu/mcas/2003/release_na/g8math.pdf #22)

2) **2** **In the diagram below, $\angle 1 \cong \angle 4$.**



Which of the following conclusions does *not* have to be true?

A $\angle 3$ and $\angle 4$ are supplementary angles.

B Line l is parallel to line m.

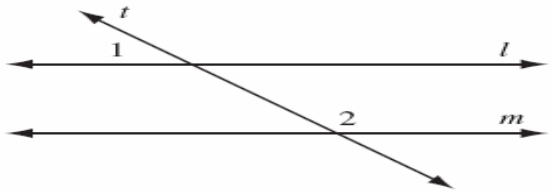
C $\angle 1 \cong \angle 3$

D $\angle 2 \cong \angle 3$

(California Standards Test, 2005 Released Test Questions, http://www.cde.ca.gov/ta/tg/sr/documents/rtqgeom.pdf #2)

3)

In the accompanying diagram, parallel lines l and m are cut by transversal t.



Which statement about angles 1 and 2 *must* be true?

A $\angle 1 \cong \angle 2$.

B $\angle 1$ is the complement of $\angle 2$.

C $\angle 1$ is the supplement of $\angle 2$.

D $\angle 1$ and $\angle 2$ are right angles.

(California Standards Test, 2005 Released Test Questions, http://www.cde.ca.gov/ta/tg/sr/documents/rtqgeom.pdf #16)

Resources:

Textbook Materials:

Connected Mathematics, Grade 6, Shapes and Designs, (entire unit).
Connected Mathematics, Grade 8, Kaleidoscopes, Hubcaps and Mirrors, Investigation 2.
Springboard Mathematics Middle School I, The College Board, What's My Name?
Springboard Mathematics Geometry, The College Board, Patios by Madeline.

Supplementary Materials:

Larson, Boswell, Stiff, *Geometry: Concepts and Skills*, McDougal Littell, Chapter 3, 2004 (DCPS Geometry text that explains angle relationships, parallel and perpendicular lines.)
http://www.ies.co.jp/math/products/geo1/applets/kakuhei/kakuhei.html (A website with an interactive Java applet allowing students to manipulate parallel lines and transversals to see resulting angle pairs.)
http://argyll.epsb.ca/jreed/javaMath/geometry/altint.html (A website that allows manipulation of a Geometer's Sketchpad sketch including simultaneous angle measurement.)

Grade 8 Unit 5 Standards-Based Worksheet

District of Columbia Public Schools – Mathematics

STANDARD

8.G.5. Apply spatial reasoning by recognizing and drawing two-dimensional representations of three-dimensional objects (e.g., nets, projections, and perspective drawings of cylinders, prisms, and cones).

Concepts:

- spatial reasoning
- two-dimensional representations
- three-dimensional objects
- nets
- projections
- perspective drawings
- cylinders
- prisms
- cones

Skills:

- apply (spatial reasoning)
- recognizing (two-dimensional representations of three-dimensional objects)
- drawing (two-dimensional representations of three-dimensional objects)

Big Ideas:

- It is practical and possible to represent three-dimensional objects in two-dimensional form.
- Two-dimensional drawings, while taking different forms, all reveal dimensions of width, length, and height.
- Technology helps to model three-dimensional objects in two-dimensions.

Essential Questions:

- Why do we need to represent three-dimensional solids in two-dimensions?
- How are the different ways to represent solids in 2-dimensions similar? How are they different?
- How is perspective created in two-dimensional drawings of three-dimensional objects?

Engaging Scenario:

You are very comfortable with computers and are teaching a "how to" class about computers at a senior citizen center near your home. One of the topics that seniors have asked about is "computer assisted design", since they have seen numerous movies in which the graphics were partially or fully computer-generated. You decide to teach them about it with a very simple demonstration of alternative methods of creating a two-dimensional representation of a rectangular prism: a net, a traditional projection, a perspective drawing, and a projection suitable for computer assisted design (CAD).

Plan a lesson for the seniors that include making a model of a rectangular prism, making a net for the prism, connecting traditional projections and the projections needed for CAD, and breaking down the steps of perspective drawings. You know that if you teach them well, the seniors will leave the class appreciating how CAD is easy to understand in the context of other methods of representing three-dimensional objects.

PERFORMANCE TASKS	THINKING SKILLS (LOT → HOT)*	PERFORMANCE TASK ASSESSMENT (PROFICIENT CRITERIA)
Students identify rectangular prisms found in "real life".	identify (knowledge) LOT	Students identify boxes that hold food or other merchandise, office buildings, dressers or other furniture, refrigerators or other appliances, and so on.
Students convert a box brought to class into a net. (Any small corrugated cardboard box will suffice, and containers for sticks of butter or margarine are ideal.) They will use this net as a model for further tasks.	convert (comprehension) LOT	Students convert the box to a net by removing sections that are used for reinforcement but not part of the surface faces.
Students create two original nets for a rectangular prism using their model as a guide.	create (synthesis) HOT	Students create nets with the correct number and size of surface faces to form a rectangular prism; Students may or may not include "tabs" for connecting faces.
Students use one of their nets to label faces for an orthographic projection and cut away the three faces that they determine as necessary for the projection.	determine (application) LOT	Students will typically label the four larger faces as "bottom view", "front view", "top view", and "rear view", with the smaller faces labeled ""left side view" and "right side view". The three faces that should remain for a traditional projection (as commonly see in architectural blueprints) are the top, front, and rear side views.
Students compare their projection with the information needed to produce a simple CAD drawing.	compare (analysis) HOT	Students determine that a CAD drawing will add perspective to a two-dimensional face. Since opposite faces are congruent, they only need the distances between those faces to create a projection with the help of a computer. In a traditional projection, they need three views of the solid.

Students breakdown the steps for making a standard geometric drawing of a rectangular prism.	breakdown (analysis) HOT	Student steps should include the following for standard drawings: ▪ Drawing two *identical* rectangles, one above and not touching the second; the upper rectangle should be just to the right or left of the lower rectangle's midline. ▪ Drawing four diagonal lines connecting the vertices of the two vertical planes.
Students compare these steps with those of making a perspective drawing.	compare (evaluation) HOT	Students' steps for the comparison with perspective drawing need only note the need for the two rectangles to be of *different* sizes, with the larger one lower than the smaller one. Sizing them in this way will create the "vanishing point" needed in perspective drawings.

*See Bloom's Taxonomy of Thinking Skills to determine higher order thinking skills (HOT) or lower order thinking skills (LOT). The goal is to create tasks that employ higher order thinking skills.

Standard Assessment:

1) What is the minimum number of congruent, equilateral triangles needed to construct a three-dimensional figure if no other shapes are used?
   a. 3
   b. 4
   c. 6
   d. 8

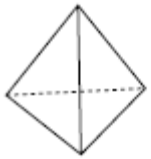
(MCAS, 2004 Mathematics-Grade8, Question #13)

2) The net for a tetrahedron would be composed of:
   a. 4 triangular faces
   b. 3 triangular faces and a square face
   c. 4 triangular faces and a square face
   d. 2 triangular faces and 3 square faces

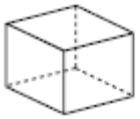
3) Mei Ling gave the following description of a three-dimensional figure.
   - The solid has 6 faces.
   - The solid has 8 vertices.
   - The solid has 12 edges.

Which of the following figures matches Mei Ling's description?

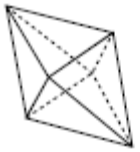
A.



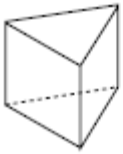
B.



C.



D.



(MCAS, 2003 Mathematics-Grade8, Question #16)

Resources:

Textbook Materials:

Connected Mathematics, Grade 7, Prentice Hall, Building Boxes, Investigation 1; Finding Volumes of Boxes, Investigation 3; Cylinders, Investigation 4; Cones and Spheres, Investigation 5.

Springboard Mathematics Middle School Math II, The College Board, All Boxed Up.

Supplementary Materials:

http://mathforum.org/sum95/math_and/perspective/perspect.html (An introduction to perspective drawing.)

http://www.senteacher.org/wk/3dshape.php (This website has ten printable nets for geometric solids.)

http://www.emachineshop.com/demo/index.htm (A simple example of how a front view projection of a disk is transformed into a three-dimensional gear by CAD.)

Grade 8 Unit 5 Standards-Based Worksheet

District of Columbia Public Schools — Mathematics

STANDARD

8.G.6. Find the distance between two points on a coordinate plane using the distance formula; find the midpoint of the line segment; recognize that the distance formula is an application of the Pythagorean theorem.

Concepts:

- points
- coordinate plane
- distance formula
- midpoint
- line segment
- Pythagorean theorem

Skills:

- find (distance between two points on coordinate plane; midpoint of line segment)
- recognize (distance formula is an application of Pythagorean theorem)

Big Ideas:

- Geometric figures and relationships can be represented in the coordinate plane.
- Geometric figures can be measured in the coordinate plane.
- The distance formula is derived from the Pythagorean theorem.

Essential Questions:

- Why is it important to learn about geometric figures and relationships in the coordinate plane?
- How does the distance formula relate to the Pythagorean theorem?
- How do you represent geometric figures in the coordinate plane?
- How do you measure geometric figures using the coordinate plane?

Engaging Scenario:

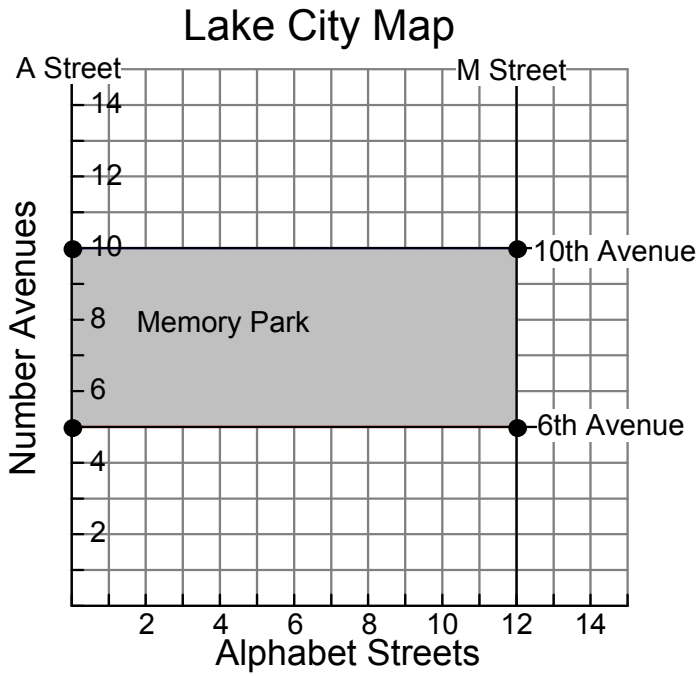
You are visiting your cousin in Lake City during your summer vacation. After touring a museum you are resting at the southwest corner of Memory Park near the intersection of A Street and 6th Avenue. You receive a phone call from your cousin who is at the corner of M Street and 10th Avenue. Your cousin asks you to cut across the park and meet her at a point halfway between your two locations. She also wants to know how far that point is from each of you "as the crow flies". Identify you and your cousin's coordinates.

Draw a line that represents the distance between your locations "as the crow flies". What is the midpoint of the line segment?

Each block is 500 feet.

1. How far do you have to travel to meet your cousin?

2. How much distance do you and your cousin save by cutting through the middle of Memory Park instead of walking down the streets that border the park to meet one another?



PERFORMANCE TASKS	THINKING SKILLS (LOT → HOT)*	PERFORMANCE TASK ASSESSMENT (PROFICIENT CRITERIA)
Students label the locations as ordered pairs.	label (knowledge) LOT	The students correctly label the locations as the ordered pairs (0, 5) and (12, 10).
Students construct a line that represents the distance 'as the crow flies'.	construct (application) LOT	Students construct a line segment connecting (0, 5) and (12, 10).
Students calculate the midpoint location (ordered pair).	calculate (application) LOT	Students calculate the midpoint (using the midpoint formula) to be (6, 7.5). The grid distance to the midpoint (using the distance formula and dividing the distance by 2) is 6.32 units. Students should multiply this figure by 500 feet to obtain a distance on the ground of 3,162 feet.

Students devise a method to calculate the actual distance traveled.	devise (synthesis) HOT	Students devise a method using the distance formula (or Pythagorean theorem) to find the distance each person travels. Students may find the distance between the two original points and then divide it by two. They may also find the distance between one endpoint and the midpoint. A complete solution will take into account that the each block is 500 feet. The distance is 6.5 blocks or 3250 feet.
Students assess how much distance is saved by cutting across the park.	assess (evaluation) HOT	Student assessments should conclude that cutting across the park saves four blocks or 2000 feet total so each person walks 1000 less feet if they meet by cutting across the park.

*See Bloom's Taxonomy of Thinking Skills to determine higher order thinking skills (HOT) or lower order thinking skills (LOT). The goal is to create tasks that employ higher order thinking skills.

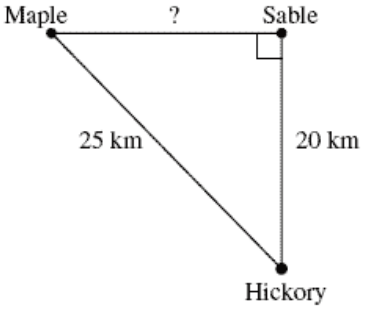
Standard Assessment:

1) *AB* has one endpoint at *A*(2, 5), and its midpoint is at (4, 0). What are the coordinates of B the other endpoint of segment *AB*?

   a. (2, –5) b. (3, 2.5) c. (6, –5) d. (6, 2.5)

(MCAS, 2005 Mathematics- Grade 10, Question # 25)

2) The roads connecting the three towns on the map below form a right triangle. Two of the distances are given.



Based on the distances given on the map, what is the distance between Maple and Sable?

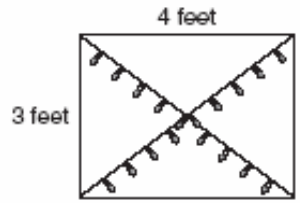
   a. 12 km b. 15 km c. 16 km d. 19 km

(*MCAS, 2005 Mathematics - Grade 8, Question #30)*

3) Mrs. Gonzalez wants to string lights along both diagonals of a rectangular window as shown. What is the minimum length of lights Mrs. Gonzalez will need?

a. 5 ft
b. 7 ft
c. 10 ft
d. 14 ft

(TAKS, 2006 Mathematics-Grade 8, Question #33)



Resources:

Textbook Materials:

Connected Mathematics, Grade 8, Looking for Pythagoras, Finding Areas and Lengths, Investigation 2; Investigation 3.

Springboard Mathematics Middle School II, The College Board, Stop the Presses.

Supplementary Materials:

http://www.netsoc.tcd.ie/~jgilbert/maths_site/applets/line_geometry (Links to "Line Geometry – The Basic Formulae" by Daniel Kelleher for applets on distance and midpoint formulas.)

http://physci.kennesaw.edu/javamirror/ntnujava/abc/Pythagoras.html (An interactive website for demonstrating the proof of the Pythagorean Theorem.)

http://www.shodor.org/interactivate/lessons/PythagoreanTheorem/ (An interactive website with lessons on computing lengths, areas and perimeters in the coordinate plane.)

Grade 8 Unit 5 Standards-Based Worksheet

District of Columbia Public Schools — Mathematics

STANDARD

8.M.2. Understand the concept of surface area and volume; given the formulas, determine the surface area and volume of rectangular prisms, cylinders, and spheres.

Concepts:
- surface area (of rectangular prisms, cylinders, spheres)
- volumes (of rectangular prisms, cylinders, spheres)

Skills:
- understand (concept of surface area and volume)
- determine (surface area and volume)

Big Ideas:
- Surface area is a two-dimensional measurement; volume is a three-dimensional measurement.
- Surface areas of prisms and cylinders are sums of areas.
- Volumes of prisms and cylinders are products of areas and heights.

Essential Questions:
- Why do we need to measure surface area and volume of various solids?
- How are nets and surface areas related?
- Why is volume usually a more practical measurement than surface area?
- Why are units of measurement squared for surface area and cubed for volume?
- How do I identify necessary dimensions for formulas for surface area and volume?

Engaging Scenario:
You are a swimming pool contractor who builds and maintains indoor pools. You've recently taken over management of a new indoor wading pool used solely by small children. You visit the pool and note these dimensions:

1. The pool is a rectangle, 25 meters in length and 10 meters in width. It has a constant depth of ½ meter. At one end of the pool, there is a semi-circular step, 1/6 meter high and with radius of 1 meter. A top-view is shown to the right.

2. The pool is in poor condition and needs to have its interior surface repainted. You will need to determine the surface area in square feet to purchase an adequate amount of paint. (One meter = 3.28 feet.)

3. You also need to determine the pool's volume so that you can order adequate supplies of chlorine and other chemicals to keep the water safe.

Write up a bid for this job. Include the total surface area that needs to be repainted, the volume of the pool so you can estimate the amount of chemicals, and a scale drawing of the pool.

PERFORMANCE TASKS	THINKING SKILLS (LOT → HOT)*	PERFORMANCE TASK ASSESSMENT (PROFICIENT CRITERIA)
Students convert the dimensions of the pool from metric units to units of feet.	convert (comprehension) LOT	Students convert the linear measures using 1 meter = 3.28 feet. The pool's dimensions should be calculated as: Length: 82 feet Width: 32.8 feet Height: 1.64 feet Height of step: 0.55 feet Radius of step: 3.28 feet Student answers may vary slightly due to rounding errors.
Students develop a scale drawing of the pool using graph paper.	develop (application) LOT	Students develop a scale drawing of the pool with the dimensions of the pool clearly labeled. The dimensions should be to scale.
Students plan how to compute the surface area of the pool. (This is best done in a cooperative group.)	plan (synthesis) HOT	Students plan how to compute the surface area of the interior of the pool. They should consider the fact that there is: • A rectangular prism of dimensions 82 feet by 32.8 feet, with only a bottom base. • The additional surface area from a steps: the difference between the area of a rectangular cross-section of a cylinder (the area of the step attached to the wall) and half the lateral area of the cylinder (the "riser" of the step).

Students compute the surface area of the pool using their plan. (Best done in a cooperative group.)	compute (application) LOT	Students compute the surface area using their plan. Student answers should be close to 3,068.2 sq. ft. and may vary slightly due to rounding errors. A complete solution will show how students arrived at their result, not just the final answer.
Students plan how to determine the volume of the pool and then compute it. (Best done in a cooperative group.)	plan (synthesis) HOT compute (application) LOT	Students plan a method that incorporates the fact that the volume of the pool is reduced by the volume of the step. Students should compute it as the volume of 82 feet by 32.8 feet by 1.64 feet prism *minus* the volume of a half-cylinder (the step). Student solutions should be approximately 4,401.65 cubic feet.
Students summarize their work in written form.	summarize (evaluation) HOT	Student summaries include their scale drawing and detailed solution methods to both the surface area and volume of the pool. Answers may vary due to rounding differences, but should clearly take into account how the step influences the surface area and volume of the pool.

*See Bloom's Taxonomy of Thinking Skills to determine higher order thinking skills (HOT) or lower order thinking skills (LOT). The goal is to create tasks that employ higher order thinking skills.

Standard Assessment:
1)

Lea made two candles in the shape of right rectangular prisms. The first candle is 15 cm high, 8 cm long, and 8 cm wide. The second candle is 5 cm higher but has the same length and width. How much additional wax was needed to make the taller candle?

A $320\ cm^3$

B $640\ cm^3$

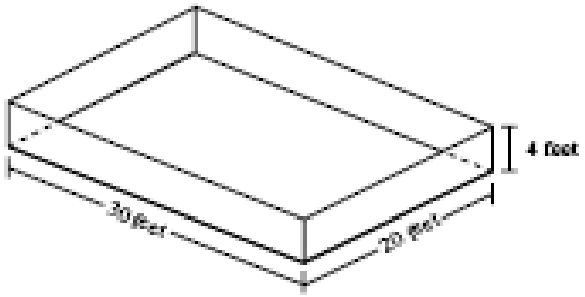
C $960\ cm^3$

D $1280\ cm^3$

(CST, 2005 Mathematics-Grade 10, Question #25)

2)

Alain works for a company that built a goldfish pond for a local university. He has to plaster the interior sides of the pond, which is shaped like a rectangular prism with the dimensions shown in the picture below.



a. What is the volume, in cubic feet, of the goldfish pond? Show or explain your work.

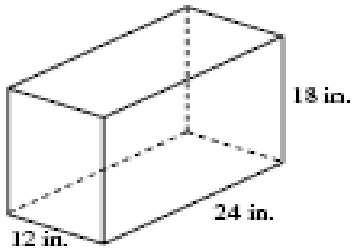
b. What is the total surface area of the interior sides and bottom of the pond? Show or explain your work.

c. If the company charges $1.50 per square foot to plaster the pond, what will it cost to plaster the 4 interior sides and the bottom of the pond?

(MCAS, 2003 Mathematics-Grade8, Question #39)

3)

The figure below shows an aquarium that is shaped like a rectangular prism.



a. What is the volume, in cubic inches, of the aquarium?

b. One gallon is equal to 231 cubic inches. How many gallons of water will the aquarium hold?

c. If 10 gallons of water were poured into the empty aquarium, what would be the depth, in inches, of the water? Show your work or explain how you got your answer.

(MCAS, 2004 Mathematics-Grade8, Question #39)

Resources:

Textbook Materials:

Connected Mathematics, Grade 7, Filling and Wrapping, Investigations 1, 2, 4-7.
Connected Mathematics, Grade 8, Say It With Symbols, Investigation 5.
Springboard Mathematics Middle School Math II, The College Board, All Boxed Up.

Supplementary Materials:

http://www.math.com/tables/geometry/surfareas.htm (A website containing all relevant formulas for surface area and volume.)
http://www.shodor.org/interactivate/activities/SurfaceAreaAndVolume/ (An applet that shows the interaction between changing dimensions and the resulting volume and surface area.)
http://www.mste.uiuc.edu/exner/java.f/cubemove/default.html (An applet that improves spatial visualization skills by allowing the viewer to fill a prism with cubes.)
Larson, Boswell, Stiff, Geometry: Concepts and Skills, McDougal Littell, 2004, Chapter 9 - Surface Area and Volume (DCPS textbook resource providing more information on the standard.)

Grade 8 Unit 5 Standards-Based Worksheet

District of Columbia Public Schools — Mathematics

STANDARD

8.M.4. Solve problems about similar figures and scale drawings. Understand that when the lengths of all dimensions of an object are multiplied by a scale factor, the surface area is multiplied by the square of the scale factor and the volume is multiplied by the cube of the scale factor.

Concepts:

- similar figures
- scale drawings
- lengths
- dimensions
- object
- scale factor
- surface area
- multiply
- square
- volume
- cube

Skills:

- solve problems (about similar figures and scale drawings)
- understand (how scale factor affects surface area and volume of similar solids)

Big Ideas:

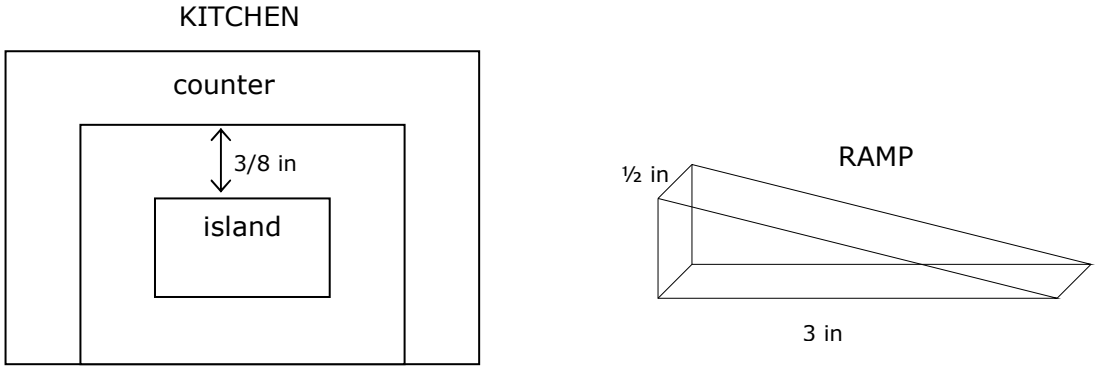
- Scale drawings allow us to grasp and manipulate spatial relationships among even the largest objects.
- Recognition of similarity in *linear* dimensions of figures can provide power in calculations in *all* dimensions.
- A scale drawing is always similar to the original figure.

Essential Questions:

- Why do we use scale drawings?
- How is the scale factor calculated for similar figures?
- Why is the ratio of linear dimensions for two similar figures the same as the ratio of the scale factor?
- Why are the ratios of the area dimensions of similar solids the same as the ratio of the square of the scale factor?
- Why is the ratio of the volume of similar solids the same as the ratio of the cube of the scale factor?

Engaging Scenario:
You are a contractor, specializing in remodeling existing homes that are both handicapped accessible and meet indoor air quality standards. On your current project, you are concerned that blueprints for the kitchen and an access ramp may not reflect the needs of a wheelchair user in the family. You also need to make sure that a recently installed ventilator is large enough to handle the air volume in the home. (All the air in a house should be changed at least one time every three hours.)

The family provides you with blueprints with a scale factor of 1in to 96in. The kitchen blueprint shows 3/8 inches of space between the center island and all counters. The blueprints also show an access ramp that will be built alongside a garage. On the blueprint, the ramp is ½ inch wide and extends for three inches along the side wall of the garage.



Notes on the blueprint reveal that the house has a total area of 0.34 square feet and ceilings on a blue print side-view are 0.08 feet above the floor. The actual ramp begins at the top of a porch, which is 18 inches above the ground. Current regulations for handicapped access require a 60 inch diameter circle for a wheelchair to turn in place. They allow access ramps to rise 30 inches in 30 feet (a scale of 1 to 12).

Using this information:

- Determine if a proposed wheelchair ramp to the porch has too steep a rise.
- Determine if there is adequate "turnaround" space in the kitchen for a wheelchair user in the family.
- Compute the volume of air in the home and assess the adequacy of a ventilator that moves 145 cubic feet of air per minute.

PERFORMANCE TASKS	THINKING SKILLS (LOT → HOT)*	PERFORMANCE TASK ASSESSMENT (PROFICIENT CRITERIA)
Students calculate the real dimensions of the ramp using the scale factor.	calculate (application) LOT	Students use the one to 96 scale factor to calculate that the ramp is 288 inches (24 feet) long and 48 inches (4 feet) wide.

Students analyze whether the ramp is too steep.	analyze (analysis) HOT	Students' analysis should conclude that with a run ratio of one to 12, a rise of 18 inches must have a run of 216 inches. With a run of 288 inches, the rise will not be too steep for the run. They should conclude that the ramp meets regulations.
Students calculate the real dimensions of the distance between the center island in the kitchen and all counters.	calculate (application) LOT	Students calculate that a distance of 3/8 inches in the blueprint corresponds to 36 inches in the kitchen.
Students analyze whether a wheelchair could turn in place in the space between the island and the counters.	analyze (analysis) HOT	Students' analysis should conclude that with a 60 inch diameter circle needed, the wheelchair could not turn in place in a space of 36 inches.
Students formulate and implement a plan for using the area of the scale blueprint and the height of ceilings on the scale blueprint to find the volume of the air in the house in cubic feet.	formulate (synthesis) HOT	Students formulate and implement a plan that first finds the ratio of volumes in the scale rectangular prism and in the rectangular prism represented by the house and then computes the corresponding volume of the house. Since the scale factor for linear dimensions is one to 96, the ratio of the volumes is 1^3 to 96^3. The volume in the scale drawing is .03 cubic feet. Using this scale factor the house volume is 26,542 cubic feet.
Students assess whether a 145 cubic feet per minute ventilator can replace 26,542 cubic feet of air in three hours.	assess (evaluation) HOT	Students' assessments are based on converting the rate of 145cfm to 8700 cubic feet per hour, and then to 26,100 cubic feet per three hours, and then comparing this rate to the volume of the house (26,542 cubic feet). They should conclude that the ventilator is slightly undersized.

*See Bloom's Taxonomy of Thinking Skills to determine higher order thinking skills (HOT) or lower order thinking skills (LOT). The goal is to create tasks that employ higher order thinking skills.

Standard Assessment:

1) The perimeters of two squares are in a ratio of 4 to 9. What is the ratio between the areas of the two squares?
    a. 2 to 3
    b. 4 to 9
    c. 16 to 27
    d. 16 to 81

(CST, 2005 Mathematics-Grade 10, Question #24)

2) Chan is designing a new swimming pool that will have a length of 34 feet. He plans to make a scale drawing of the pool. In his drawing, 1/4 inch represents 1 foot. What should be the length, in inches, of Chan's scale drawing of the pool?

(MCAS, 2005 Mathematics-Grade8, Question #20)

3) A cylindrical tank has a radius equal to 6 centimeters. The tank has a height of 16 centimeters (1 milliliter = 1 cubic centimeter).
    a. What is the radius of the base, in centimeters?
    b. What is the volume of the cylinder in milliliters? Show your work.
    c. If both the radius and the height of the cylinder were doubled, what would be the volume of the cylinder in milliliters? Show your work.
    d. Based on your answers to parts b and c, what is the ratio of the volume of the smaller tank to the volume of the larger tank? Show your work.

(MCAS, 2002 Mathematics-Grade 8, Question #39)

Resources:

Textbook Materials:

Connected Mathematics, Grade 7, Prentice Hall, Comparing and Scaling, Investigation 3; Stretching and Shrinking, Investigations 1-6.

Springboard Mathematics Middle School II, The College Board, All Boxed Up.

Supplementary Materials:

Larson, Boswell, Stiff, *Geometry: Concepts and Skills*, McDougal Littell, 2004, Chapter 7 (Similarity), Chapter 9 (Surface Area and Volume) (DCPS Geometry text provides additional information about this standard.)

http://library.thinkquest.org/C006354/12_5.html (Illustration of basic conditions for use of squared scale factor and cubed scale factor.)

http://standards.nctm.org/document/eexamples/chap6/6.3/index.htm (NCTM-created applet: increasing the size of one of two similar figures changes ratios of perimeters and areas.)